BARRIER SEARCH

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Monterey, California

THESIS

BARRIER SEARCH

by

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Barrier Search

by

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Submitted in partial fulfillment of the
requirements for the degree of

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Simulation generated probabilities for an Anti-Submarine Barrier composed of submarines are examined from a statistical viewpoint. A probabilistic model which is not generally considered to be applicable to this case is demonstrated to be statistically supported. The applicability of the model is justified probabilistically. A statistical estimating relationship is then developed to estimate the sole input parameter from submarine FIGURE-OF-MERIT.

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LIST OF SYMBOLS AND ABBREVIATIONS

SYMBOL	MEANING
C	The frontal width of the operating area of the barrier submarine.
D	The frontal width of the entire submarine barrier, composed of one or more operating areas of width C.
P(x)	The lateral range curve. The probability of detecting a target which passes in a straight line encounter at lateral range x.
u	Target speed
v	Searcher speed
V_B	Barrier submarine speed
V_T	Transiting submarine speed
W	Kill sweep width
w	Relative speed of the searcher
W_B	Kill sweep width for the barrier submarine
W_T	Kill sweep width for the transiting submarine

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I. INTRODUCTION

A. THE TACTICAL PROBLEM

The encounter treated is that of an Anti-Submarine Warfare barrier consisting of submarines. Each barrier submarine is confined to an operating area, hereafter called a cell, due to the requirement to remain in a safe haven. The cell is part of a larger barrier across a restricted passage of width D. The transiting submarine is obliged to pass through the passage, and hence the barrier. The barrier has uniform cells and may be composed of several cells in depth. All detections are made with passive SONAR. No external intelligence of the position of either opposing force is allowed. All detections are made by the submarines and all kills are made without the aid of external forces. Transiting submarines are alone, and the time between transits is sufficiently long to prevent there being any effect of one transit upon another. If cells are arranged in sequence along the transit track, they are sufficiently separated to prevent there being any effect of one engagement upon another (This assumption is relaxed for purposes of investigation in Section II.C). The barrier is sufficiently wide (D is sufficiently large) to make any special effects from the cells on the ends negligible.

By virtue of the above assumptions, the encounter may be studied as a single cell with a single transiting submarine,

and a single barrier submarine. The barrier in total may then be constructed probabilistically from the building block of one cell. Figure (1) is a geographical diagram of one cell in the barrier, with pertinent variables labelled.

B. THE SIMULATION

For the purposes of simulation, the following assumptions were deemed appropriate:

(i) The transiting submarine chooses a penetration point uniformly along the cell front, since the limits of the cell are unknown to it.

(ii) The transiting submarine proceeds at transit speed (V_T) until detecting the barrier submarine, at which time it slows to quietest speed to evade.

(iii) The transiting submarine penetrates the cell front perpendicularly, and continues on a straight track throughout.

(iv) No counter measures are employed.

(v) The barrier submarine proceeds at patrol speed (V_B) across its cell until a detection, at which time it commences interception tactics. For a short initial period it steers the target's bearing, after which time sufficient information is available for a firm track. When firm track is established, the barrier submarine proceeds to the intercept point.

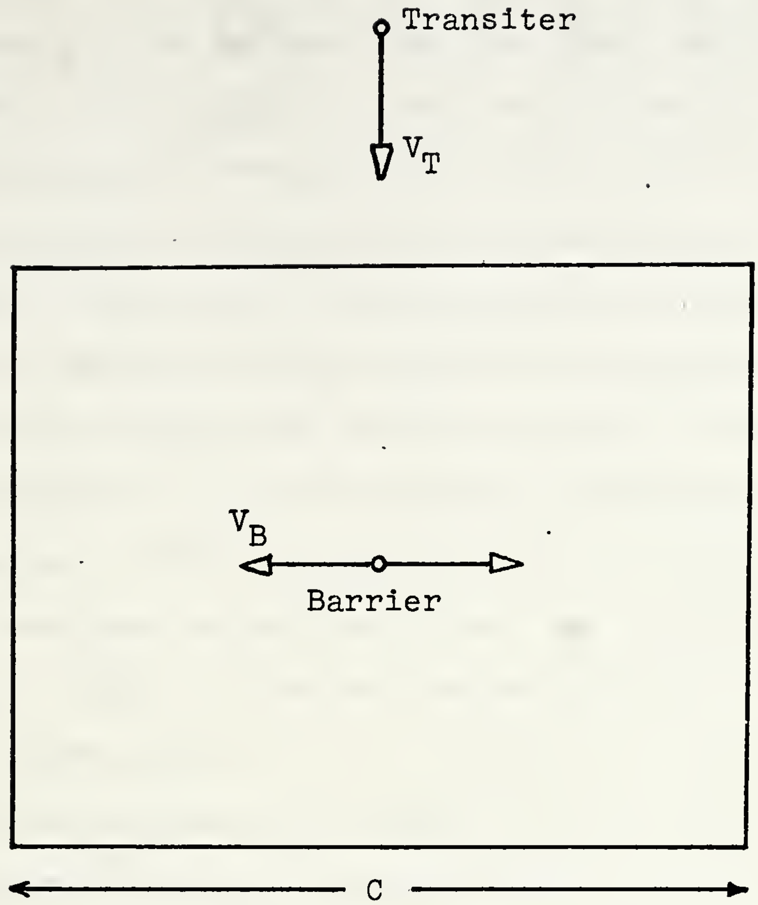


Figure 1

The Tactical Scenario

The simulation was by digital computer. It serves the purpose of providing data for analysis only, and is described here to give insight into the likely characteristics of the data. The validity of the simulation is not a subject of direct analysis. The ability of the data to support reasonable models, and internal consistency were the facets of validity tested. The simulation was constructed and run by the Center for Naval Analyses and should be considered separate from the analysis contained herein.

The simulation is a time-stepped, event-store, Monte-Carlo program. Each stochastic process is modeled stochastically, and each probabilistic event is determined by random number. Input parameters include acoustic characteristics of both submarines, sea conditions, initial velocities of both submarines and weapons load of both submarines. Sea conditions were not varied for any simulation runs.

Two hundred and fifty replications of each submarine pairing were conducted. Taking any particular binary value output by the simulation to be a bernoulli random variable (i.e.: target killed or not killed), the variance of any probability estimator is

$$\mathrm{Var}\ (\hat{p}) = \frac{p(1 - p)}{n}$$

This quantity is maximized for a p of .5, so the maximum possible variance of such output is .001.

With two hundred and fifty replications the Central Limit Theorem will be assumed to hold, and hence the estimators will be assumed to be distributed as gaussian random variables.

C. THE ANALYTICAL PROBLEM

The original analysis of the tactical problem was conducted by simulation. Seven distinct barrier submarine hull and sonar combinations and four distinct transiter hull and sonar combinations were included.

Various quantities were compiled and computed from the simulation run, most for the purpose of checking consistency. The sole parameters finally desired from this simulation were the probabilities of kill for the barrier and the transiting submarines.

The number of distinct simulations is expressed as the product of (i) the number of cell widths sufficiently close to allow interpolation and sufficiently numerous to avoid extrapolation; (ii) the number of opposing velocity pairs; (iii) the product of the numbers of distinct opposing submarines. The essence of the problem is thus to reduce the information contained in such output to a tractable analytical expression. The purpose of this reduction is four-fold. First, the information must be made convenient for a decision maker. Second, the information should be expressed as a function of tactical parameters and characteristics to allow inspection of the dynamics of the

problem and to assess validity. Third, the information should be expressed as a function of input parameters and decision variables to enable the comparison of alternative courses of action. Fourth, an acceptable analytical model can either reduce or eliminate the need for further simulation computer time, should additional range of input variables be desired.

II. THE SEARCH MODEL

The following discussion treats the searcher as moving and the target as stationary, in order to be in consonance with discussions in the literature. Clearly, the interchangeability of searcher and target may be used to achieve a desired result.

A common model in search theory is the Random Search Model, now a classic in the field [see ref. 1]. The assumptions for this search law to hold are listed in ref. 1 as

(i) The target (here the searcher) is uniformly distributed within A.

(ii) The observer's (here "the target's") path is random in A in the sense of as having its different (not too near) portions placed independently of one another in A.

(iii) On any portion of the path which is small relatively to the total length of path but decidedly larger than the range of possible detection, the observer always detects the target (here "the target is always detected by the searcher") within the lateral range W/2 on either side of the path and never beyond. This last assumption can be relaxed to the extent that a number W (search sweep width) may be defined by integrating over the lateral range curve P(x) where

$$P(x) = P(\text{detect a target passing at lateral range } x)$$

A searcher with lateral range curve P(x) will detect as many targets in the entire plane as one with a definite range law detector such as indicated by assumption (iii), as long as the targets are distributed uniformly, and as long as

$$\int_{-\infty}^{\infty} P(x)\,dx = W$$

where W is the diameter of the definite search law detector.

Under the Random Search Law,

$$P(\text{detection}) = 1 - e^{-W\ell/A}$$

W = sweep width
ℓ = track length
A = area containing target

If the searcher transits through A then

$$\frac{\ell}{A} = \frac{1}{C}$$

C = width of A

thus

$$P(\text{detection}) = 1 = e^{-W/C}$$

For the purposes of this analysis, the assumptions of the Random Search Law hold, with the exception of (ii). The searcher (target in this context) is not searching along a random track, but is actually proceeding directly along an effectively zig-zag non-overlapping track relative to the target.

A second model is developed in ref. 1 which may also be applied. This model is a barrier model. It will be referred to here as the Channel Patrol Model.

Suppose a searcher patrols at speed v across a channel of width c. Then one patrol (here a channel crossing) is completed in $\frac{c}{v}$ hours. The target is approaching at speed u straight down the channel. During one patrol of the searcher he travels $\frac{c}{v}$ u miles. Relative to the target, the searcher is traveling a zig-zag path at relative speed w, where

$$w = \sqrt{u^2 + v^2}$$

(see Figure 2).

Thus the total area within which the target may be found in one patrol is

$$A = (\text{height})(\text{base}) = (u \frac{c}{v})\ c$$

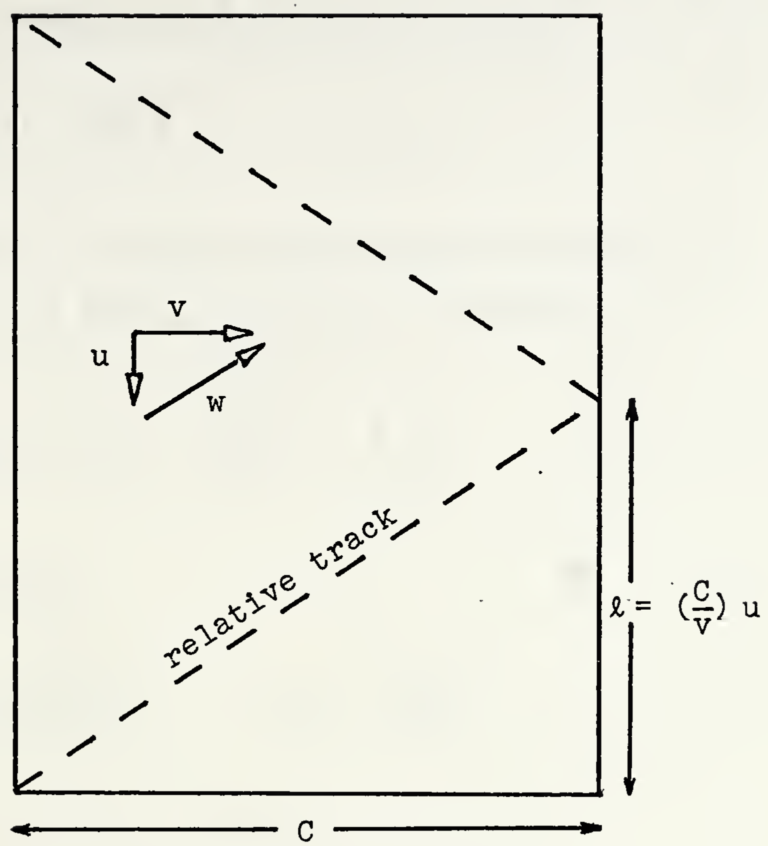


Figure 2

The Channel Patrol Model

The total relative area swept by the searcher in one patrol is

$$B = (\text{velocity})(\text{time})(\text{sweep width})$$

$$= (w)\ (\tfrac{c}{v})\ W$$

If the target is located within this relative area uniformly, then the probability of detection is

$$P(\text{det}) = \frac{B}{A} = \frac{w\ (\frac{c}{v})\ W}{u\ (\frac{c}{v})\ C} = \frac{W}{C}\ \frac{w}{u}$$

Noting that

$$w = \sqrt{u^2 + v^2} = u\sqrt{1 + (\tfrac{v}{u})^2}$$

then

$$P(\text{det}) = \frac{W}{C}\sqrt{1 + (\tfrac{v}{u})^2} = \frac{W}{C}\ k$$

if $\frac{W}{C}\ k << 1$ then approximately

$$P(\text{det}) = 1 - e^{-\frac{W}{C}k}$$

This approximation involves some reduction in the probability of kill, since

$$1 - e^{-k\frac{W}{C}} < k\ \frac{W}{C}$$

This may be justified in that the constrictions of a cell or channel will deny the barrier submarine the portion of his sensor's range which is beyond the limits of his cell. This phenomenon will be called clipping, and will be discussed in Appendix B.

If the probability of kill given detection is $\rlap{/}p$, for all targets then

$$P(\text{kill}) = 1 - e^{-\rlap{/}p \frac{W}{C} k}$$

$k = 1$ random search model

$k = \sqrt{1 + (\frac{v}{u})^2}$ channel patrol model

The quantities $\rlap{/}p$ and k are statistically nuisance parameters, and may be suppressed by defining a new quantity, kill sweep width, and also denoted W, so that

(1) $$P(\text{kill}) = 1 - e^{-\frac{W}{C}}$$

W = kill sweep width

The advantage of this representation is that p is no longer needed, provided that kill sweep width can be determined, as will be the case.

B. MODEL VERIFICATION

Rather than questioning the reasonableness of the assumptions, or the applicability of the model to the tactical problem, an alternative approach seems fruitful. Let us suppose the applicability of the model and then test the supposition statistically.

It is natural to suppose that W (sweep width) is a function of at least (a) speed of both submarines and (b) quietness of both submarines. It is explicitly not a function of C (cell width). The obvious test of the Random Search Model is to invert the formula, injecting known C and monte-carlo estimates of P(kill) to obtain W estimates, and then test the variability of w with respect to C. Now,

$$p_k = 1 - e^{-\frac{W}{C}}$$

therefore

$$W = -C \log_e (1 - p_k)$$

Both a W_T and W_B are found, which are those associated with the transiter and barrier respectively.

From considerations of the data, not included here, it is immediately obvious that the variation in W with respect to C is very small. This impression was verified statistically using analysis of variance. It was found that the

variations in W were justifiably attributed to statistical fluctuation in the monte-carlo estimation of P(kill), and not to any systematic model inconsistencies. The detection/kill model is thus supported statistically (see statistical appendix) in the sense that it fails to be contradicted.

A traditional approach to the simulation data used here probably involve the estimation of W from technological considerations, and a comparison of this computed value and the attendant probabilities to the results of the simulation. The approach taken herein is to take the simulation results and derive the implied values of W. Validation of the model would then involve consistency within the model. The main reason for this approach is that only kill probabilities are desired, whereas typical estimation of w is for <u>search</u> sweep width. Thus inversion of the procedure allows for elimination of the necessity of analytically solving the kinematical problem of kill, counter-detection and evasion, which is the primary source of difficulty in the traditional approach. Additionally, should the predicted sweep width be sufficiently different from an experimentally derived value, then the classical approach offers little recourse except to discard the predicted sweep width, the experimental data or the model.

C. IMPLICATIONS OF THE MODEL

There are very important tactical considerations which might argue for various barrier configurations for submarines. Two building block configurations are here examined to determine their probabilistic advisability. The first is the "stacked" barrier, with submarines deployed in depth. The second is the "packed" barrier with submarines deployed along a line (see Fig. 3).

Now, suppose that there is some larger barrier of width D in which these cells are placed. Alternative I is to place one packed barrier, alternative II is to stack k less dense barriers with the same number of submarines, across the channel D.

Let

n = number of submarines in the barrier

p_1 = P(kill a transiter under I)

p_2 = P(kill a transiter under II)

$q_1 = 1 - p_1$

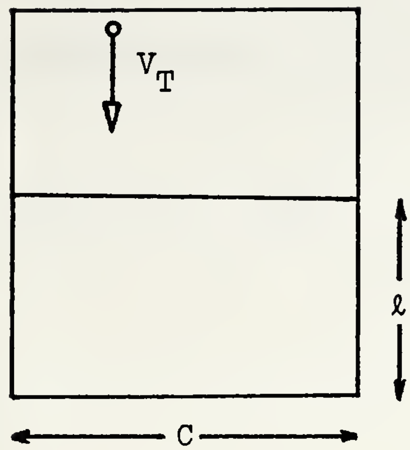
$q_2 = 1 - p_2$

Now, cell width for any single line barrier is related to n and D by

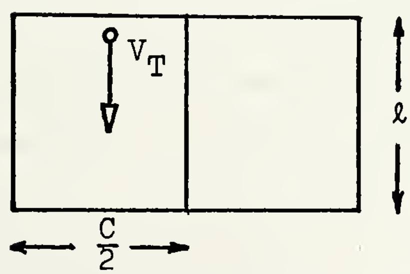
$$C = \frac{D}{n}$$

so

$$1 - p_1 = q_1 = e^{-\frac{W}{C}} = e^{-\frac{Wn}{D}}$$



Alternative I
The Stacked Barrier



Alternative II
The Packed Barrier

Figure 3

The Packed and Stacked Barriers

and assuming independence among stacked barriers

$$1 - p_2 = q_2 = e^{-\frac{Wn_1}{D}} e^{-\frac{Wn_2}{D}} \cdots e^{-\frac{Wn_k}{D}}$$

where there are k stacked barriers, each with n_i submarines $(i = 1,\ldots,k)$. Now

$$1 - p_2 = q_2 = e^{-\frac{W}{D} \Sigma n_i} = e^{-\frac{Wn}{D}} = q_1 = 1 - p_1$$

and

$$p_1 = p_2 \; .$$

Thus there is nothing probabilistic to distinguish between the two barriers under independence.

Assuming, on the other hand, that there exists some dependence among the k stacked barriers, then either type of dependence is possible. The type of dependence results from the nature of the opposing forces. There are only four possible results from a transit across the barrier. They are the following states:

1 Barrier killed transiter survives
2 Transiter killed barrier survives
3 Both submarines killed
4 No submarines killed

In the case of 2 and 3, the type of dependence between the barrier in question and subsequent barriers is unimportant since the transiter submarine has been killed, and thus will not encounter any subsequent barrier.

In the case of 1, it is likely that the combat between the two submarines would alert the next barrier, or possibly several barriers, and allow for maximum readiness and optimum placing. In the case of 4, whatever special circumstances allowed the transiter to evade the first barrier would be likely to carry over into the second barrier. These arguments are expressed analytically as follows. Let

$$P_{ij} = P(\text{enter state } j \mid \text{leave state } i)$$

$$i = 1,2,3,4$$
$$j = 1,2,3,4$$

where one state transition occurs upon transiting one barrier. Then in these cases

$$P_{12} > P_{1j} \qquad j = 1,3,4$$

and

$$P_{44} > P_{4j} \qquad j = 1,2,3$$

Remembering that states 1 and 4 correspond to transiter not killed, and states 2 and 3 correspond to transiter killed it can be seen that if the first encounter results in state 4, then the transiter will be most likely to avoid being killed by a state 4 result in subsequent barriers. If the first encounter results in state 1, then

the transiter is more likely to be killed by a state 2 result in subsequent barriers. Thus, depending upon the likely nature of the first encounter, the decision can be made whether to stack or pack barriers.

III. PARAMETRIC PREDICTION OF SWEEP WIDTH

A. DESCRIPTION OF INPUTS

The applicability of the Random Search Model being established, the next step is to find a method for predicting the value of W in Eq. (1). Since the analysis is intended to serve as an aid to decision making, it is desirable to use variables in the prediction whose relationship to W is simple, and which are under the control of the decision-maker, or tactical commander.

In submarine warfare, an important measure of effectiveness of a submarine is its acoustic quietness, called FIGURE-OF-MERIT (FOM). FOM measures the self noise of the submarine, as well as the signal level of the opponent. It combines these quantities into one, expressing the quietness of the submarine relative to its opponent.

The FOM used in this paper is as follows:

$$FOM = L_s - L_e - DT$$

$$L_s = 10 \log_{10}(10^{L_{sm}/10} + 10^{L_{sv}/10})$$

$$L_e = 10 \log_{10}(10^{L_{ea}/10} + 10^{L_{em}/10} + 10^{L_{ev}/10})$$

L_s = Radiated Noise

L_{sm} = radiated machinery noise

L_{sv} = radiated velocity noise

L_e = Self Noise

L_{ea} = ambient self noise

L_{em} = machinery self noise

L_{ev} = velocity self noise

DT = Detection Threshold

Velocities for computation were taken as V_B, the barrier patrol speed, and V_T, the transit speed. After detection, velocities changed for both submarines, but no account of this was taken, since this represented mainly kinematics.

B. THE ESTIMATING RELATIONSHIP

The dependent variables to be predicted were W_T and W_B (sweep width of transiter and barrier, respectively). Independent variables were assumed to be FOM_B and FOM_T (FIGURE OF MERIT Barrier and Transiter, respectively). A linear relationship was found to be the best predictor of Kill Sweep Width, with form:

$$(2) \quad \begin{aligned} W_T &= a + b\, FOM_T + d\, FOM_B \qquad a,b,d,f,g,h > 0 \\ W_B &= f - g\, FOM_T + h\, FOM_B \end{aligned}$$

Both kill sweep widths were tested, that for the barrier (W_B) and the transiter (W_T), using stepwise linear regression. In each case, it was found that the most significant variable for predicting sweep width was the

FOM of the submarine to whom the W belonged. That is, the most significant variable in predicting W_T was FOM_T. Both FOM's were found to be significant for both sweep widths.

The coefficients preceding the variables in Eq. (2) were all as might reasonably be expected, except for d, the coefficient of FOM_B. It would seem that the sign of d should be negative, so that improvement in the performance of the barrier submarine would degrade the performance of the transiter. This would ordinarily be the case, except for the implications of the tactical assumptions. Since the transiter does not seek combat, all opportunity for such combat arises by way of the barrier detecting the transiter. Thus, as the barrier improves in performance, more opportunity to attack arises for the transiter.

In order to ensure adequacy of the regression, two additional tests were conducted. For each, the difference between the predicted values of sweep width and the actual values, called residuals, were found. These residuals were tested using Analysis of Variance.

The first test concerned the hull types. The question tested was whether there was any information associated with the Hull/Sonar combinations which was not conveyed by means of FIGURE OF MERIT. The results indicated that there was no reason to believe that any additional information about the submarine was likely to be of any use, beyond FIGURE OF MERIT.

The second test was whether the patrol speed (V_B) or the transit speed (V_T) had any effect beyond that conveyed through FOM. The results indicated that the patrol speed of the barrier submarine was not significant, whereas the transit speed (V_T) was significant. This is due to two factors. First, the faster the transiter travels, the less time he is exposed to detection. Second, the transiter is less susceptible to intercept at higher speed. This remaining effect due to V_T is a drawback in the model, since the best possible prediction is not being carried out. In view of the excellent fit of the linear regression (see the Statistical Appendix) the model seems adequate for most needs. It is possible that the additional complica-aion of the model or the increased need for model inputs would make a more refined model undesirable.

C. A SECONDARY EFFECT OF THE ANALYSIS

In the course of the statistical analysis of the simulation data, an important result was obtained, above and beyond the requirements of data compactness and manageability. The result was the detection, from statistical considerations, of programming errors in the simulation. Tactical simulations are always large, in a computer code sense, and difficult to completely "de-bug". In this instance, a logic control variable had been included, to allow changing of torpedo P(kill) during a run. The program was set up initially using dummy P(kill)'s

for all torpedoes. When the final data runs were made, the logic control variable was set to inject the correct P(kill)'s for barrier types 1 through 4, but was erroneously omitted for barrier types 5 through 7. The omission was not noticed until the statistical analysis began. As a part of the statistical analysis, the residuals (Actual W minus predicted W) were plotted for all submarine types. It was noticed that barrier types 1 through 4 were all performing poorer than predicted, and barrier types 5 through 7 were performing better than predicted. This discontinuity was not due simply to different Hull/Sonar types, because one Hull type and one Sonar were found in both groups, although not together. When the programmer saw this effect he correctly deduced the error and re-ran the entire set of barrier types 5 through 7. The resulting residuals were again plotted, and no longer displayed the discontinuity. It is unlikely that the error would ever have been discovered without the statistical analysis.

IV. CONCLUSIONS

The philosophy pursued herein, that a model is valid if it is statistically supported is a profitable philosophy only insofar as it aids analysis. If a model is reasonable from an analytical point of view and also supportable statistically, then the best solution has been arrived at. It is the case here, as in many instances, that the model chosen was not uniquely applicable, nor were its requisite assumptions completely satisfied. Hopefully, any doubts about applicability were dispelled by the compelling statistical evidence.

The method of inversion of results to obtain unknown inputs, used here to determine W, is also somewhat unorthodox. The resultant estimates of inputs are naturally only as good as the original results. Assuming an acceptable level of validity, these inverted quantities may be compared to engineering or similar estimates, probably to the benefit of both the engineering and tactical analyst. Perfectly valid and useful models are no doubt frequently rejected because the improper estimation of input quantities causes inexplicable differences between actual and predicted results. It seems reasonable, however, that statistically verifiable consistency in quantities inverted by way of a model would argue in favor of the model, and cause re-examination of the engineering estimate in case of disagreement. Conversely, models which are supposed to be

invalid due to assumptions being clearly violated but which are found to be statistically supportable can often be extended to include the case in question by further considerations of the problem. The phenomenon of clipping developed here is an example of this potential.

APPENDIX A

STATISTICAL APPENDIX

1. Validation of the Model

The formula

$$p = P(\text{kill}) = 1 - e^{-\frac{W}{C}}$$

W = kill sweep width
C = cell width

was inverted to yield

$$\hat{W} = -C \ln (1 - \hat{p})$$

which then becomes a maximum likelihood estimator of W.

The hypotheses were

$$H_0: \quad p = 1 - e^{-\frac{W}{C}}$$

$$H_1: \quad H_0 \text{ is false}$$

or equivalently

$$H_0: \quad W = - c \ln (1 - p)$$

$$H_1: \quad H_0 \text{ is false}$$

The test was by analysis of variance using a Completely Randomized Factorial design with treatments of barrier type, transiter type and cell width at seven, four and three levels, respectively. Submarine types were taken as fixed factors, cell width as a random factor. Two separate tests were conducted, one for W_B and one for W_T (barrier and transiter kill sweep widths, respectively). F ratios and significances were as tabulated in Tables I and II. Based on the significance of the cell width treatment, the null hypothesis was accepted at significance level of .05.

2. Parametric Prediction of Sweep Width

The estimating relationships

$$W_T = a + b\ FOM_T + d\ FOM_B$$

$$W_B = f - g\ FOM_T + h\ FOM_B$$

were tested by linear regression. The test statistics for W_T were:

$$F = 8.9$$
$$R^2 = .787$$
$$n = 168$$

The regression was thus significant at the level of .05. The t statistics associated with the constants b and d were 18.714 and 2.98 respectively, both significant at .05.

TABLE I. ANALYSIS OF VARIANCE OF CELL WIDTH FOR W_T

Treatment	Degrees of Freedom	F Ratio	Significance
Barrier	6	80.1	a
Transiter	3	153.9	a
C	2	2.015	b

TABLE II. ANALYSIS OF VARIANCE OF CELL WIDTH FOR W_B

Treatment	Degrees of Freedom	F Ratio	Significance
Barrier	6	76.94	a
Transiter	3	181.621	a
C	2	1.6515	b

Notes:

a $\ll$ .001

b $>$ .25

In each regression, data included each of twenty-eight submarine pairings at two separate values of V_B and three separate values of V_T making a total of 168 data points. In each regression there were no other variables ever tested as independent variables, thus the significance level reported is valid.

Upon completion of each regression, the residuals were tested for any variability remaining which could be removed. This was done by considering two analyses of variance of each set of residuals.

The first post-regression analysis of variance used barrier and transiter hull/sonar type as treatments at 7 and 4 levels respectively. The hypotheses were

H_0: Hull/Sonar type has no effect on the residual

H_1: H_0 false

Test design was Randomized Block Factorial Design with a block consisting of the six distinct velocity pairings. The two treatments were considered as fixed. F ratios and significances were as tabulated in Tables III and IV. Based on the resulting significances, the null hypothesis was accepted. The overall significance was .05. This implies an individual significance of .013, where

$$P(\text{No test value outside critical region}) = (1-\alpha)^4$$

thus, $$(1-\alpha)^4 = .95$$

$$\alpha = 1 - (.95)^{\frac{1}{4}} = .013$$

TABLE III. ANALYSIS OF VARIANCE OF RESIDUALS OF W_T (SUBMARINE-TYPE TREATMENTS)

Treatment	Degrees of Freedom	F Ratio	Significance
Barrier	6	.48	.8173
Transiter	3	1.04	.3991

TABLE IV. ANALYSIS OF VARIANCE OF RESIDUALS OF W_B (SUBMARINE-TYPE TREATMENTS)

Treatment	Degrees of Freedom	F Ratio	Significance
Barrier	6	.6	.7267
Transiter	3	4.13	.0216

The second post-regression analysis of variance used V_B and V_T as treatments at two and three levels respectively. The hypotheses were:

H_0: V_B has no effect on the residuals

H_1: H_0 false

and

H_0: V_T has no effect on the residuals

H_1: H_0 false

For this experiment, there were four separate tests, one for each unit's velocity, and thus two for each regression. The likelihood is that one or more of these null hypotheses might be true with no implications for the others. Experiment design was again Randomized Block Factorial with each effect now taken as random. Each block consisted of 28 distinct submarine pairings. F ratios and significance levels are tabulated in Tables V and VI. Based on the resulting significances, the velocity of the transiting submarine is significant for the probability of its detection and kill by the barrier submarine. This result is probably due to the resulting reduced time available to the barrier submarine to detect, and to the increased difficulty in closing with and killing given detection.

TABLE V. ANALYSIS OF VARIANCE OF RESIDUALS OF W_T (VELOCITY TREATMENTS)

Treatment	Degrees of Freedom	F Ratio	Significance
V_B	1	3.5	.20
V_T	2	7.4	.12

TABLE VI. ANALYSIS OF VARIANCE OF RESIDUALS OF W_B (VELOCITY TREATMENTS)

Treatment	Degrees of Freedom	F Ratio	Significance
V_B	1	6.1	.13
V_T	2	24.7	.039

APPENDIX B

THE CLIPPING PHENOMENON

Suppose that a searcher has Lateral Range Curve p(x) (vanishing after R) the probability of detecting a target passing at lateral range x. Define

$$W = 2 \int_0^R p(x)\, dx$$

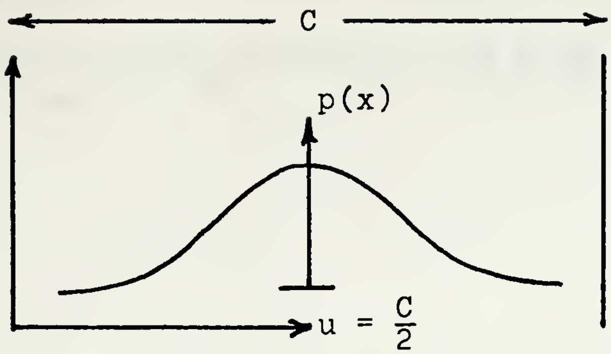
Then if the target is uniformly distributed across width C, centered at the observer,

$$P(\text{detect}) = \frac{W}{C} \qquad\qquad R < C$$

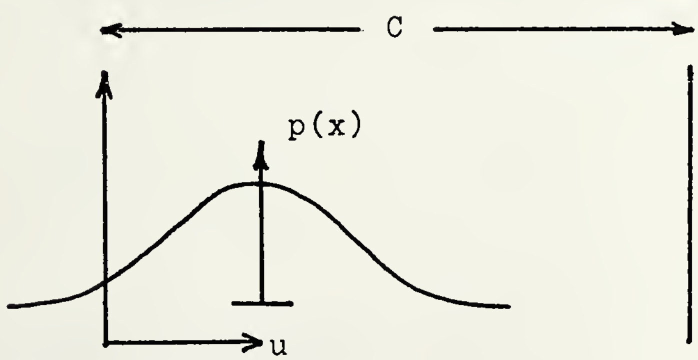
Now, however, suppose that the curve p(x) vanishes outside a range R greater than C, and that the searcher is constrained to remain within width C and thus may not pursue nor take any action with respect to a target outside of C. Let the searcher's position from the edge of the channel be denoted u (Figure 4).

In this case, the probability of detecting a target with lateral range (closest point of approach) distributed uniformly across C, for a searcher at u is

$$P(\text{det}) = \int_0^u p(x)\, \frac{1}{C}\, dx \; + \; \int_0^{C-u} p(x)\, \frac{1}{C}\, dx$$



No clipping, searcher centered in channel



Clipping, searcher at u

Figure 4

Clipping

denoted hereafter as

$$P(\text{det}) = \frac{1}{C}\{P(u) - P(C-u)\} = \frac{1}{C} W_e(u) \leq \frac{1}{C} W$$

where

$$P(u) = \int_0^u p(x)\, dx$$

Now, if the searcher is in turn uniformly distributed in position u across C, the probability of detection is

$$P(\text{det}) = \frac{1}{C} \int_0^C P(x) - P(c-x)\, dx = \frac{1}{C} W_a$$

Now, if the searcher is located in the center of the channel,

$$u = \frac{C}{2}$$

and so

$$P(\text{det}) = \frac{1}{C} W_e \left(\frac{C}{2}\right)$$

Clearly, if p(x) is decreasing in x for x positive, and symmetric then

$$\frac{W_a}{C} \leq \frac{W_e(\frac{C}{2})}{C} \leq \frac{W}{C}$$

Thus against a uniform target, probability of detection is greatest for a central position, and is diminished by

patrolling. Of course, if patrol is not conducted, the possibility arises that barrier station location may become compromised, so a patrol is clearly necessary.

Let p(x) be of the form

$$p(x) = e^{-ax} \qquad a > 0$$

which is reasonable for passive sonar detection (see Ref. 1), but intended only as an illustration. If the searcher is at $u = \frac{C}{2}$ (centered in the channel)

$$P(\text{det}) = \frac{1}{C} W_e \left(\frac{C}{2}\right) = \frac{W}{C}\left(1 - e^{-\frac{C}{W}}\right)$$

If the searcher is uniformly distributed across the channel

$$P(\text{det}) = \frac{1}{C} W_a = \frac{W}{C}\left(1 - \frac{W}{2C}\left(1 - e^{-\frac{2C}{W}}\right)\right)$$

These quantities are graphed in Fig. 5 as a function of $\frac{W}{C}$, along with the quantities

$$p = 1 - e^{-\frac{W}{C}}$$

and the Definite Range Law (Ref. 1)

$$p = \frac{W}{C} \qquad \frac{W}{C} < 1$$

$$= 1 \qquad \frac{W}{C} > 1$$

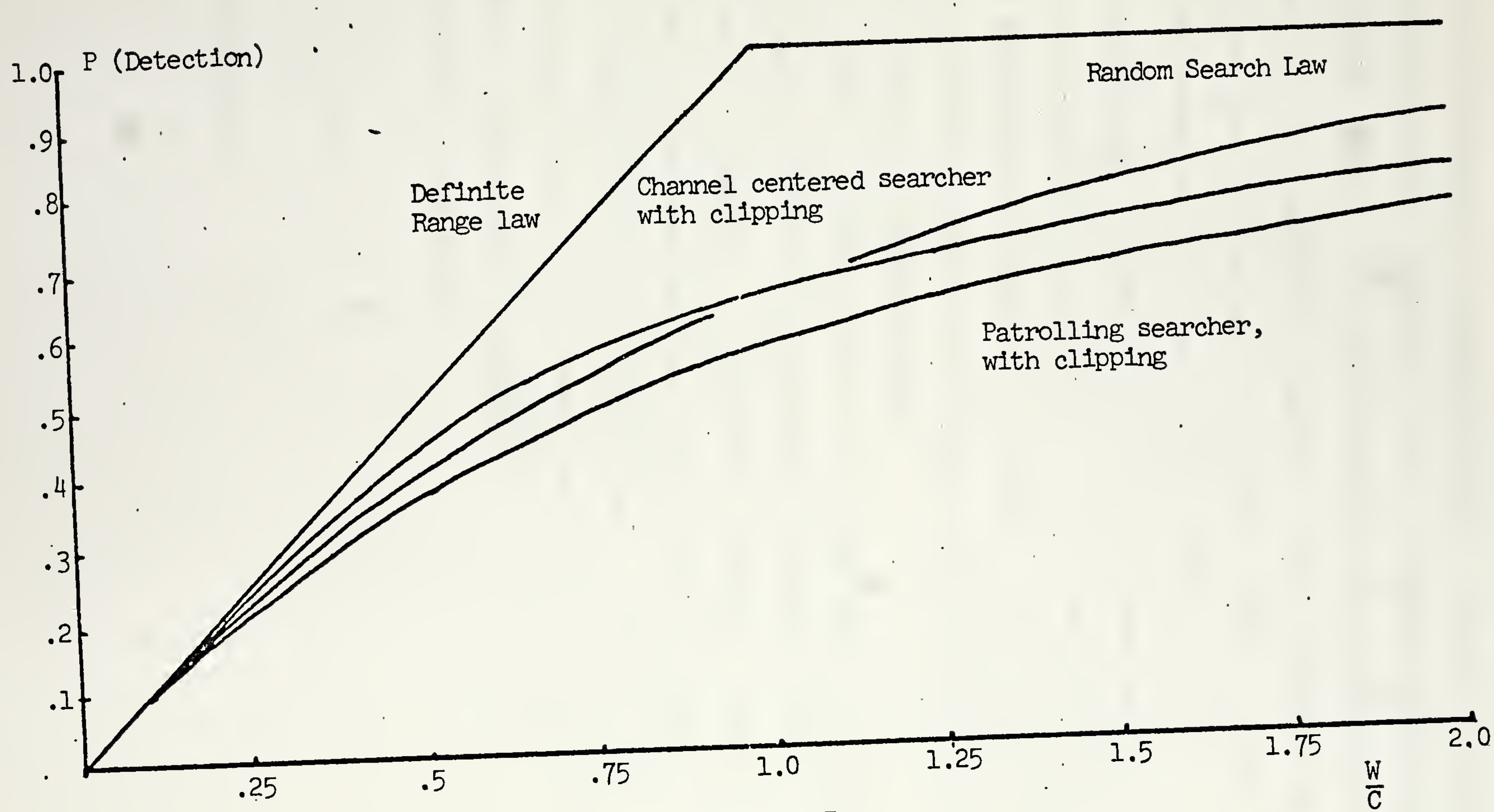


Figure 5
Comparison of Detection Probabilities

The agreement is seen to be excellent among the first three in areas of mild clipping, and when there is excessive clipping, the Random Search Law (Equation (1)) is somewhat optimistic. This contrasts sharply with the obvious inaccuracy of the last quantity, the Definite Range Law. It should now be fairly clear that the amount of clipping depends a great deal on the shape of p(x), the lateral range curve. Lacking any better information, the slight degradation implied in the Random Search Law is quite probably sufficient to a reasonable degree of accuracy. If, as in this analysis, the data support the model, then there should be no doubt as to its reasonableness.

Most cases of barrier patrol will probably be found to lie between the cases of the searcher centered in the channel and the searcher distributed uniformly across a cell. The problem treated here was within the domain of excellent agreement (moderate clipping), and hence the law should reasonably hold, since the Lateral Range Curve should have been roughly exponential in x. Thus the applicability of the formula

$$P(\text{kill}) = 1 - e^{-\frac{W}{C}}$$

is established.

LIST OF REFERENCES

1. Operations Evaluations Group Report No. 56, Search and Screening, by Bernard Osgood Koopman, p. 28, 98.

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